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IMPROVEMENTS IN THE PERTURBATION SIMULATIONS OF THE GLOBAL REFERENCE ATMOSPHERIC MODEL

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Abstract

The Global Reference Atmospheric Model (GRAM) program includes the capability for simulating pseudo-random perturbations in density, temperature, pressure, or wind components along a simulated reentry trajectory or other path through the atmosphere (e.g. a vertical profile). Some concerns have been expressed by GRAM users, however, that mean-square perturbation gradients [e.g. $\langle(\Delta f/\Delta z)^2\rangle$ for any perturbation parameter f] may be too large for small values of vertical separation Δz. The present GRAM perturbation simulations, based on a one-step autoregressive model [AR(1)], yield a power spectrum versus wavenumber k which is proportional to k^{-2} at high wavenumbers. This feature also produces mean-square perturbation differences [e.g. $\langle(\Delta f)^2\rangle$] which are directly proportional to Δz, and mean-square perturbation gradients which are inversely proportional to Δz. Thus, root-mean-square gradients, $(\Delta f/\Delta z)_{rms}$, increase with decreasing Δz as $\Delta z^{-1/2}$. This report suggests simple modifications to GRAM (e.g. changes in the form of the autoregressive correlation function used) which overcome this problem, i.e. which produce root-mean-square gradients that remain bounded as Δz approaches zero. Possible applications of more sophisticated simulation approaches, e.g. second order autoregressive models [AR(2)], or fractal model techniques, were also explored briefly but found to yield improvements which appear to be too small to justify their considerable added complexity for use in the GRAM program.

INTRODUCTION AND BACKGROUND

Figure 1 illustrates an application of the present GRAM perturbation model in simulating vertical density gradients for vertical profiles. Vertical steps of 2 km or 0.25 km were used. Larger density gradient ($\Delta f/\Delta z$) values are evident with the 0.25 km spacing.

The random perturbation model in GRAM is basically a simple Markov process, i.e. one which uses a scalar factor, rather than a transition probability matrix, to relate the current perturbation value to the previous perturbation value. In the notation of time series (Box-Jenkins) models (see, for example, Vandaele, 1983), the GRAM perturbation model is a first-order autoregressive model, or AR(1) model, with an exponential correlation function $\rho(\Delta r) = e^{-\Delta r/L}$, for spatial separation Δr. L is the integral scale of the correlation function $\rho(\Delta r)$, that is,

$$L = \int_0^\infty \rho(\Delta r)\, d\,(\Delta r) \quad . \tag{1}$$

Spectra consistent with this correlation function would be of the form (Lumley and Panofsky, 1964)

$$kF(k)/\sigma^2 = (kL/\pi)/(1 + k^2L^2) \tag{2}$$

for scalar quantities (density, temperature, etc.) and for the longitudinal spectra of vector quantities (wind components). For the transverse spectra of vector components, the spectrum would be

$$kF(k)/\sigma^2 = (kL/2\pi)(1 + 3k^2L^2)/(1 + k^2L^2)^2 \quad . \tag{3}$$

Both of these spectra vary as $F(k) \propto k^{-2}$ at large values of k ($kL >> 1$), i.e., for small scales of separation. The spectral forms are widely used in turbulence simulations, where they are referred to as the Dryden spectrum (see, for example, Fichtl, 1977, or Turner and Hill, 1982).

Figure 2 shows a comparison between the vertical spectrum of horizontal wind (equation 3) evaluated from the GRAM perturbation model and spectra presented recently by Van Zandt (1985). For wavelengths less than about 1 km (wave numbers greater than 10^{-3} cycles/m) the observed spectra are consistent with $F(k) \propto k^{-3}$, a characteristic of the spectrum of a saturated field of

gravity waves (Smith et al., 1987).

The form of the exponential correlation (and its associated Dryden spectral form) were originally selected for GRAM from studies of vertical structure function (and a limited amount of horizontal structure function) data. Examples of these earlier results (Justus and Woodrum, 1972) are shown in Figure 3. For vertical displacement, Δz, the AR(1) model used in the GRAM perturbation routine produces a structure function which is given by (Justus et al., 1986)

$$<\Delta f^2> = <[f(z+\Delta z)-f(z)]^2> = 2\,\sigma_f^2[1 - \rho(\Delta z)] \quad , \tag{4}$$

where σ_f is the standard deviation of the perturbation values. As shown by Figure 3, the structure function model provided by use of the exponential correlation $[\rho(\Delta r) = \exp(-\Delta r/L)]$ in equation (4) yields a good fit to the observed mean-square vertical differences, at least for vertical separations of 1km or larger. Structure function values of vertical separations of less than 1 km are limited by lack of vertical resolution of the Meteorological Rocket Network sensors.

For vertical displacement, Δz, the AR(1) model used in the GRAM perturbation routine produces rms gradients which, from equation (4), are given by

$$(\Delta f/\Delta z)_{rms} = \sigma_f\{2[1 - \rho(\Delta z)]\}^{1/2}/\Delta z \quad . \tag{5}$$

For the exponential correlation function, and $\Delta z/L << 1$, equation (5) can be approximated as

$$(\Delta f/\Delta z)_{rms} = \sigma_f(2/\Delta zL)^{1/2} \quad . \tag{6}$$

Thus, as the vertical step Δz is decreased in the AR(1) simulation, the rms perturbation (density or other parameter) gradient will increase inversely as the square root of Δz. This is the phenomenon illustrated in Figure 1.

Comparison of the spectra in Figure 2 and the structure function data in Figure 3 suggests that, if the vertical spacing is 1 km or greater the perturbation results will be consistent with observed spectral magnitudes and gradients. However, the fact that the model spectrum $\propto k^{-2}$ versus the observed spectrum $\propto k^{-3}$ for large k (small displacement) suggests that unrealistic gradients and spectral magnitudes might result for vertical separation of less

than 1 km. For this reason it has been recommended (Justus, et al., 1986) that simulations be done using GRAM with a minimum vertical spacing of 1 km. The results of the research reported here are intended to improve the perturbation simulations with GRAM in such a way as to allow this restriction to be relaxed.

METHODS EXAMINED FOR REVISED GRAM PERTURBATION SIMULATION

The correlation function $\rho(\Delta r)$ enters into the simulation of any perturbation function f(r) [with mean = 0 and standard deviation = σ_f] via the relation for the first-order autoregressive, AR(1), model (Vandaele, 1983)

$$f(r) = \rho(\Delta r)\ f(r-\Delta r) + \sigma_f\ [1-\rho^2(\Delta r)]^{1/2} a(r) \quad , \tag{7}$$

where a(r) is a random variable with mean = 0 and variance = 1, and Δr is the step size in successive positions. The AR(1) model produces a statistically stationary series of values for f(r) only if $\rho(n\ \Delta r) = \rho^n(\Delta r)$, that is, if $\rho(\Delta r)$ is the exponential function $\rho(\Delta r) = e^{-\Delta r/L}$ (Vandaele, 1983; Chapter 4). If the condition of statistical stationarity is relaxed, by using a correlation function other than $e^{-\Delta r/L}$, then equation (6) will not produce the desired standard deviation (σ_f) for a series of simulated values of f(r) [see, for example, Figure 4, which shows the results of using a Gaussian correlation in the AR(1) model, and for which the values of the computed standard deviation and gradients are too small at the smaller vertical separation of 0.25 km]. However, it is the exponential correlation function which imposes the k^{-2} large-wavenumber spectrum [equations (2) and (3)] and the rms-perturbation gradients which become too large as the separation Δr gets small [equation (6)].

In order to remedy this situation, one approach examined was the use of a modified correlation function which yields bounded values of rms gradients when Δr is small, hopefully without significant adverse impact on the statistical stationarity and the estimation of the proper variance for the f(r) series. If the leading terms in the expansion of $\rho(\Delta r)$ are quadratic in Δr, rather than linear in Δr as for the exponential correlation, then equation (5) would yield rms perturbations which approach a constant value as Δr approaches zero. Candidate modifications for the exponential correlation function examined include the Gaussian [$\rho(\Delta r) = \exp(-\pi\ \Delta r^2/4L^2)$], the parabolic [$\rho(\Delta r) = 1 - (2\ \Delta r/3L)^2$ for $\Delta r \leq 3L/2$; $\rho(\Delta r) = 0$ for $\Delta r > 3L/2$], and the modified exponential [$\rho(\Delta r) = 1 - a(\Delta r/L)^2$ up to some small value of Δr and $\rho(\Delta r) = \exp(-b\ \Delta r/L)$ for

larger values of Δr, with factors a and b determined from continuity of $\rho(\Delta r)$ at the transition value and the integral scale condition, equation (1)]. Other candidate correlation functions examined were $\sin(\pi\Delta x/2L)/(\pi\Delta x/2L)$ and $(1 + 2\Delta x/L)\exp(-2\Delta x/L)$.

Another approach examined was the suitability of using a two-step autoregressive model, AR(2). For this model, Vandaele (1983) gives the relation

$$f(r) = \phi_1 f(r-\Delta r) + \phi_2 f(r-2\Delta r) + \sigma_f[1 - \phi_1\rho_1 - \phi_2\rho_2]^{1/2} a(r) \ , \qquad (8)$$

where ρ_1 is the one-step autocorrelation function value, ρ_2 is the two-step autocorrelation, and the coefficients ϕ_1 and ϕ_2 are given by

$$\phi_1 = \rho_1(1 - \rho_2)/(1 - \rho_1^2) \quad , \quad \text{and}$$
$$\phi_2 = (\rho_2 - \rho_1^2)/(1 - \rho_1^2) \quad . \qquad (9)$$

The only restrictions on the correlation function $\rho(\Delta r)$ [and the one-step and two step values, ρ_1 and ρ_2, it yields] are that, for statistical stationarity, it is necessary that $\phi_1+\phi_2<1$, $\phi_2-\phi_1<1$, and $|\phi_2|<1$ (Vandaele, 1983; Chapter 4). By using equations (8), we see that all three of these conditions are met if ρ_2 is greater than ρ_1^2 or if ρ_2 satisfies the relation $2\rho_1^2-1<\rho_2<\rho_1^2$. The AR(2) model will thus be stationary for any correlation function which meets these conditions. If the correlation function violates the statistical stationarity constraint in the AR(2) model, then the term inside the square root in equation (8) becomes negative, and no simulations can be calculated by this method. If ρ_2 is equal to ρ_1^2 then the correlation function is exponential $[\rho(\Delta r)=e^{-\Delta r/L}]$ and equations (9) reduce to $\phi_1=\rho_1$ and $\phi_2=0$. Thus, the AR(2) model becomes the same as the AR(1) model if the autocorrelation function is exponential.

Another method examined for the perturbation simulations is a fractal simulation technique. For the fractal approach, a series of values for perturbations f(r) can be simulated (Carpenter, 1980) by

$$f(r) = [f(r+\Delta r) + f(r-\Delta r)]/2 + (c\ \sigma\ \Delta r/L)\ a(r) \ , \qquad (10)$$

where c is a constant (the "roughness factor"), and Δr is at first some large separation which is then successively reduced in a "recursive midpoint

reduction" process. The advantage of the fractal technique is that it can be designed to "mimic" natural curves and fluctuations quite closely. The disadvantage, in terms of application in GRAM, is that fractal simulation via equation (10) requires a "look-ahead" approach (to evaluate the $f(r+\Delta r)$ values), rather than using only one time step in the "past", as in the current AR(1) model.

RESULTS OF THE STUDY

For study of the various candidate correlation functions, a set of 10 series simulations of the equivalent of 100 km were done with vertical steps between data points of 2, 1, 0.8, 0.6, 0.4 and 0.2 km. For this test a constant value of the integral scale L was taken to be 10 km [the GRAM model actually uses height-dependent scale values and employs a two-scale model for large-scale and small-scale fluctuations (Justus, et al., 1980)]. The AR(1) model of equation (6) and the AR(2) model of equation (7) were both evaluated for each of the candidate correlation functions. The fractal model of equation (8) was also examined for a single selected value of the roughness factor c.

The correlation models used, introduced above were:

- the exponential, $\rho(\Delta r) = \exp(-\Delta r/L)$,

- the modified exponential, $\rho(\Delta r) = 1 - a(\Delta r/L)^2 \qquad \Delta r/L \le 0.05$
  $\rho(\Delta r) = \exp(-b\Delta r/L) \qquad \Delta r/L > 0.05$
  with a = 19.51615854016301 and
  b = 1.00041693941245578,

- the Gaussian, $\rho(\Delta r) = \exp(-\pi\, \Delta r^2/4L^2)$,

- the parabolic, $\rho(\Delta r) = 1 - (2\,\Delta r/3L)^2 \qquad \Delta r/L \le 1.5$
  $\rho(\Delta r) = 0 \qquad \Delta r/L > 1.5$,

- $\rho(\Delta r) = \sin(k\Delta r)/(k\Delta r)$, with $k = \pi/2L$, and

- $\rho(\Delta r) = (1+m\Delta r)\exp(-m\Delta r)$, with $m = 2/L$.

Averages and standard deviations about the averages of the ten trials are

shown in Table 1, for the observed standard deviation $\sigma(f)$ of the simulation series, and for the rms gradient value $df/dr = (\Delta f/\Delta z)_{rms}$. The input value for $\sigma(f)$ was 10% in each case, so the expected average value $\sigma(f)$ should be 10% for simulation series which are statistically stationary. Of course, as discussed above, only the exponential correlation $\rho(\Delta r) = \exp(-\Delta r/L)$ is expected to yield statistically stationary results for the first order AR(1) model.

The expected values of the rms gradient df/dr, evaluated by equation (5) are given for comparison in Table 2. All correlation functions in both the AR(1) and AR(2) models are seen to yield df/dr values close to that expected. All of the simulation approaches with all candidate correlation functions except the modified exponential are seen to yield df/dr values which are considerably below that for the present exponential correlation function, even at vertical separations larger than 1 km, where the exponential correlation is considered to yield accurate results. Only the modified exponential correlation function yields values of df/dr which are consistent with the exponential correlation results at separations larger than 1 km. The low value of df/dr for the fractal simulations could, of course, be increased by using a larger value for the roughness factor, c, than the one selected. However, the practical difficulties of implementing the fractal approach in the GRAM program, mentioned above, essentially rule out its possible application.

The results of Table 1 also show that:

- The Gaussian, parabolic, $\sin(k\Delta r)/(k\Delta r)$, and $(1+m\Delta r)\exp(-m\Delta r)$ correlation functions produce $\sigma(f)$ values which are too low, because of the statistical non-stationarity, when used in the AR(1) model.

- The modified exponential, parabolic, and $\sin(k\Delta r)/(k\Delta r)$ correlation functions are not suitable for the second order AR(2) model because of difficulties with statistical stationarity (dashes in Table 1).

- The Gaussian, parabolic and $\sin(k\Delta r)/(k\Delta r)$ correlations produce $\sigma(f)$ values which are too low (even when statistical stationarity problems are not present) when used in the second order AR(2) model.

- The $\sigma(f)$ values are about as good for the $(1+m\Delta r)\exp(-m\Delta r)$ correlation in the AR(2) model as for the exponential and modified exponential functions. However the $(1+m\Delta r)\exp(-m\Delta r)$ correlation must be ruled out for its low

resultant values of gradient, df/dr.

- Although the $\sigma(f)$ values for the fractal model results average close to the nominal value of 10.0, the computed $\sigma(f)$ values are more variable (larger ± standard deviation about the average) than for the exponential correlation.

- The $\sigma(f)$ values for the modified exponential correlation are about as good as those for the exponential correlation, when used in the AR(1) model, even in the separation range $\Delta r/L < 0.05$ where the correlation function changes to a form which no longer preserves strict statistical stationarity.

- The df/dr values from the modified exponential correlation are consistent with those from the exponential correlation for the range $\Delta r/L > 0.05$, but remain bounded by approaching a constant value for separations smaller than $\Delta r = 0.05L$, as required.

With all of these results taken into consideration, only the modified exponential correlation function in the AR(1) model meets the necessary requirements of consistency with observed $\sigma(f)$ and df/dr values at large separations ($\Delta r >$ about 1 km), while yielding bounded values of df/dr for small separations ($\Delta r <$ about 1 km). The one possible exception would be the fractal model with a larger value of the roughness factor, c. However, practical problems of implementation in GRAM rule this out without very major program modifications.

Figure 5 shows simulated vertical profiles of density perturbations computed by using the modified exponential correlation function. Comparison of Figure 5 with Figure 1, computed with the simple exponential correlation function, shows that the gradient values for the 0.25 km vertical separation case are significantly reduced, while the rms perturbation values themselves are still consistent with the input value. The results of Figures 1 and 5 (as well as Figure 4) were all computed by a two-scale perturbation model, similar to that actually used in GRAM, with a small scale value of 10 km, and a large scale value of 20 km. Input values of rms magnitude for the perturbations were 10% for both small scale and large scale perturbations (14.1% for the total perturbations shown in the Figures). Figure 5, combined with the results summarized in Table 1, thus confirm that the modified exponential correlation function adequately addresses the current problems of large gradients at small separations, while leaving the perturbation model essentially unchanged (and

consistent with observed data) at large separations.

A STUDY OF HORIZONTAL PERTURBATION STRUCTURE

During the original study of horizontal and vertical structure functions on which the GRAM correlation function and scales are based (Justus and Woodrum, 1972), only a very limited amount of data was available for horizontal structure function analysis. A more extensive data set is that of the density measurements made on Space Shuttle reentry trajectories. These data were provided to us by Mr. John. Findlay of Flight Mechanics and Control, Inc.

These shuttle data have been used in a structure function analysis by first interpolating the data, observed at irregular spacing of horizontal and vertical positions, to a constant step size in the horizontal. Since the Shuttle trajectories through the 45-95 km range analyzed are nearly horizontal (glide slope of about 1/15 or less), the rms differences between successive values of separation are taken to be representative of a horizontal structure function.

The structure function, given by equation (4), approaches the value

$$\langle \Delta f^2 \rangle = 2\ \sigma_f^2 (\Delta r/L) \qquad , \tag{10}$$

for small separations $\Delta r/L$, when the exponential correlation function applies. The observed Shuttle correlation function data were plotted on log-log scale and power-law exponents for $\langle \Delta f^2 \rangle$ versus Δr were determined by best-fit slopes. Data from overlapping height intervals of 45-65km, 55-75 km, 65-85 km and 75-95 km were considered separately. The average values of the power law exponent [expected value 1.0, from equation (6)], were found to be 1.05±0.21, 0.94±0.21, 0.95±0.25, and 1.29±0.23, respectively, for these four height ranges (± values give the one-standard deviation range about the mean value). Except for the upper height range (where the data are somewhat uncertain), these results tend to confirm the exponential correlation function form for horizontal separations as well as for the vertical separations which have been extensively examined earlier (Justus and Woodrum, 1972).

MODIFICATIONS IN THE GRAM PROGRAM CODE

The following program code changes will implement the modified correlation function model into the GRAM program. Program line number references are as given in the GRAM source code listing given in Appendix D of Justus et al. (1980).

After line CORL 45, add the following new function:

```
FUNCTION CORREL(X)
DATA A,B/19.51615854016301,1.00041693941245578/
RHO = 1./EXP(B*X)
IF(X .LT. 0.05) RHO = 1. - A*X**2
CORREL = RHO
RETURN
END
```

To replace lines PERT 40, PERT 45, PERT 50, PERT 55, PERT 60 AND PERT 65, respectively, insert the following new lines of code:

```
 10  RDS = CORREL(RDS)                                   PERT 40

 30  RTS = CORREL(RTS)                                   PERT 45

 50  RVS = CORREL(RVS)                                   PERT 50

 70  RDL = CORREL(RDL)                                   PERT 55

 90  RTL = CORREL(RTL)                                   PERT 60

110  RVL = CORREL(RVL)                                   PERT 65
```

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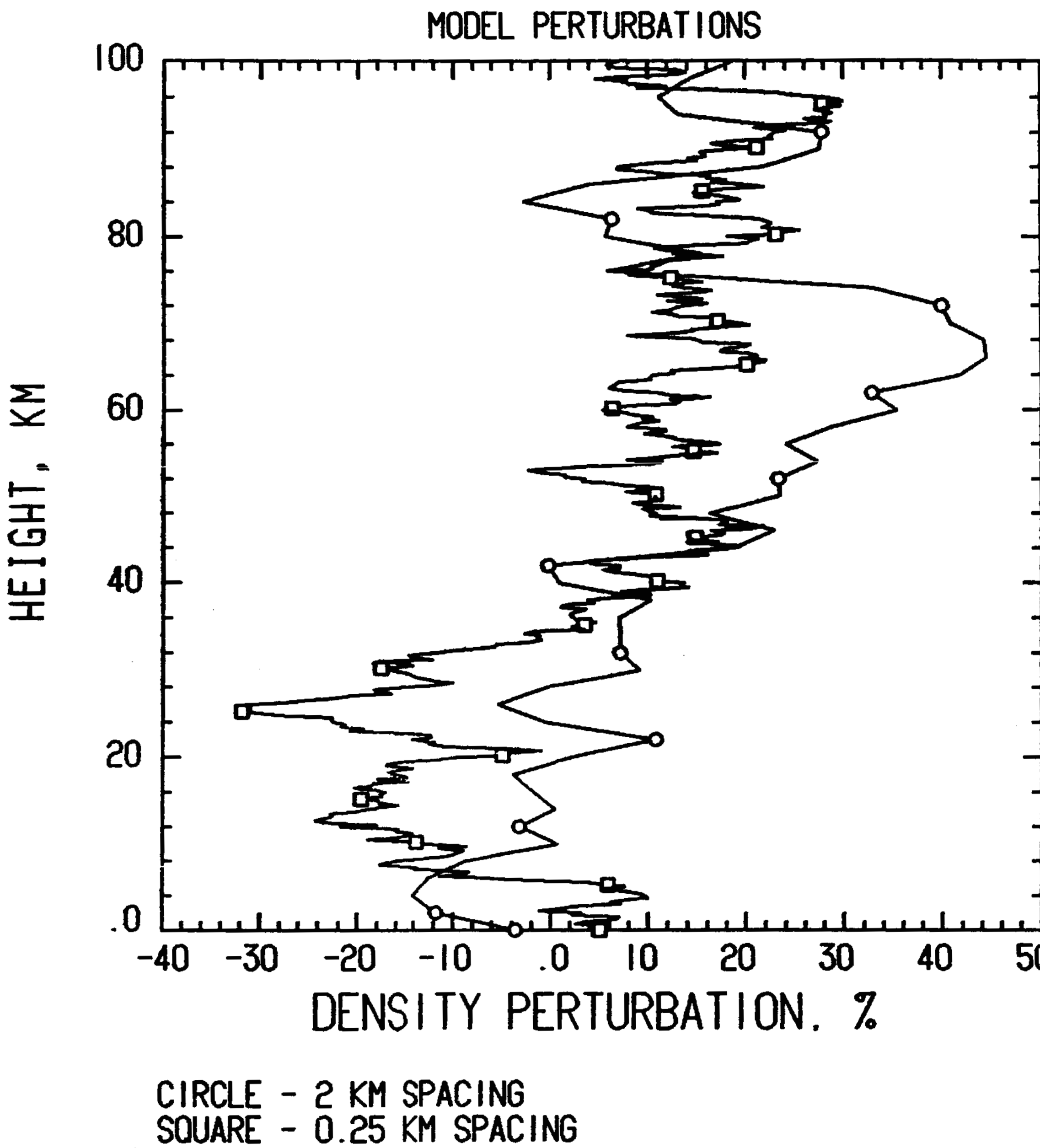

Figure 1 - Simulation of vertical profile of density perturbations with the GRAM two-scale model, with two different vertical spacings and the exponential correlation function. Small scale = 10 km, Large Scale = 20 km. Each with rms magnitude of 10% (total magnitude 14.1%)

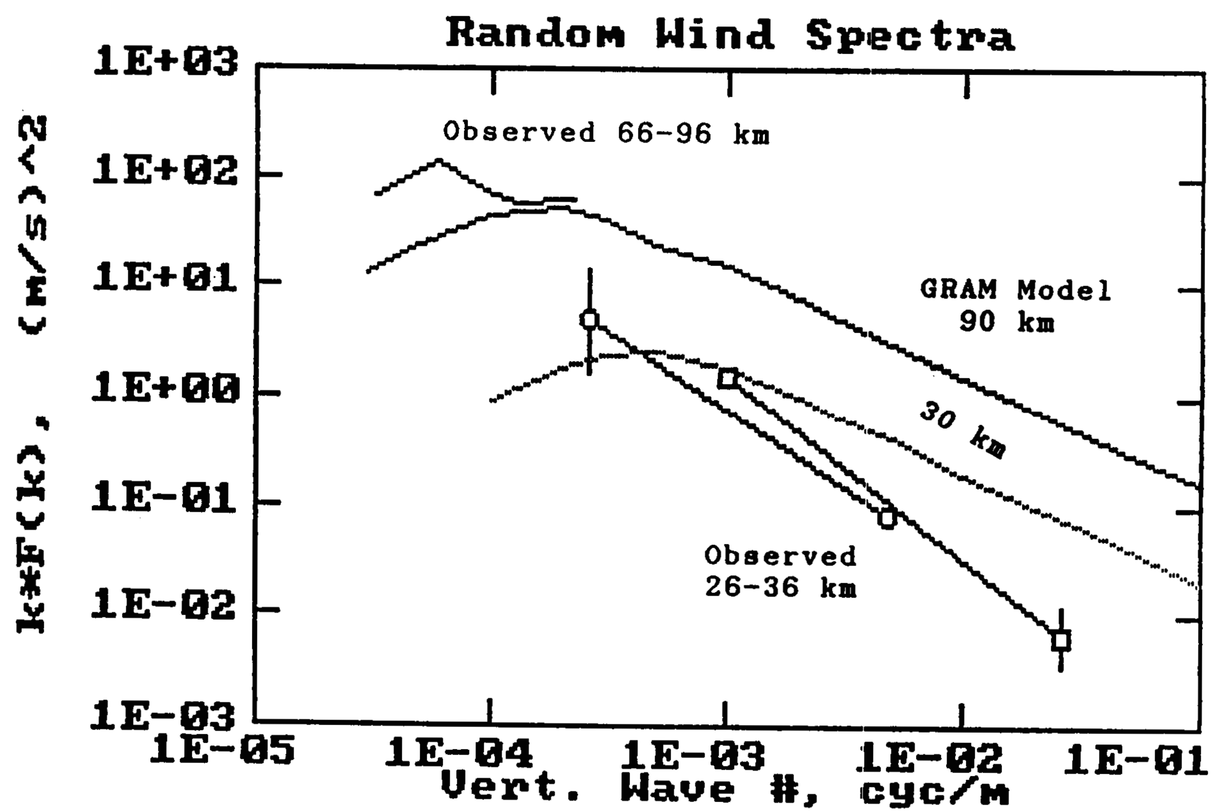

Figure 2 - Comparison of GRAM model wind spectral with observations reported by Van Zandt (1985).

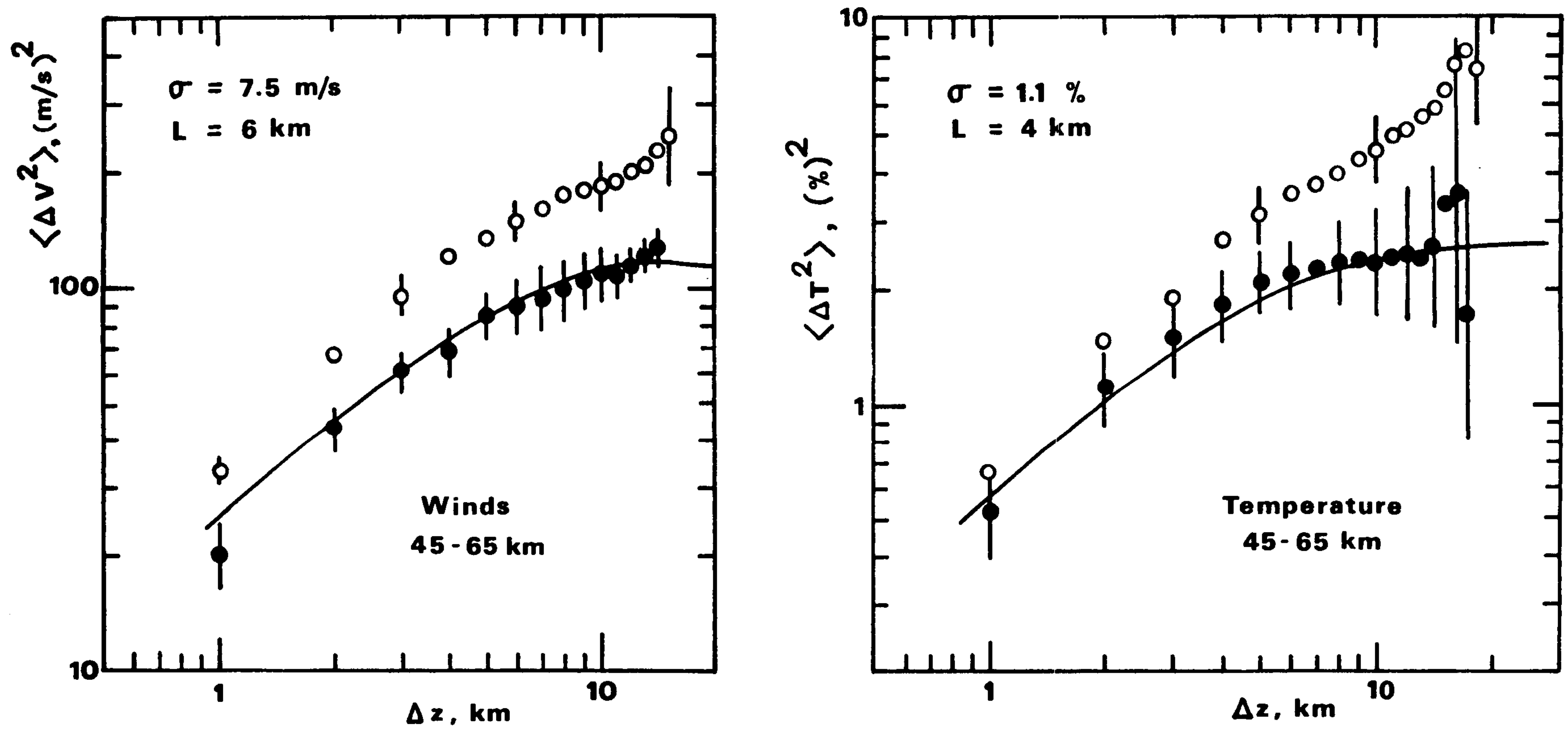

Figure 3 - Observed vertical structure function of small and large-scale wind and temperature perturbations in the 45-65 km height range (Justus and Woodrum, 1972). Curves through the data are structure functions computed by the exponential correlation function.

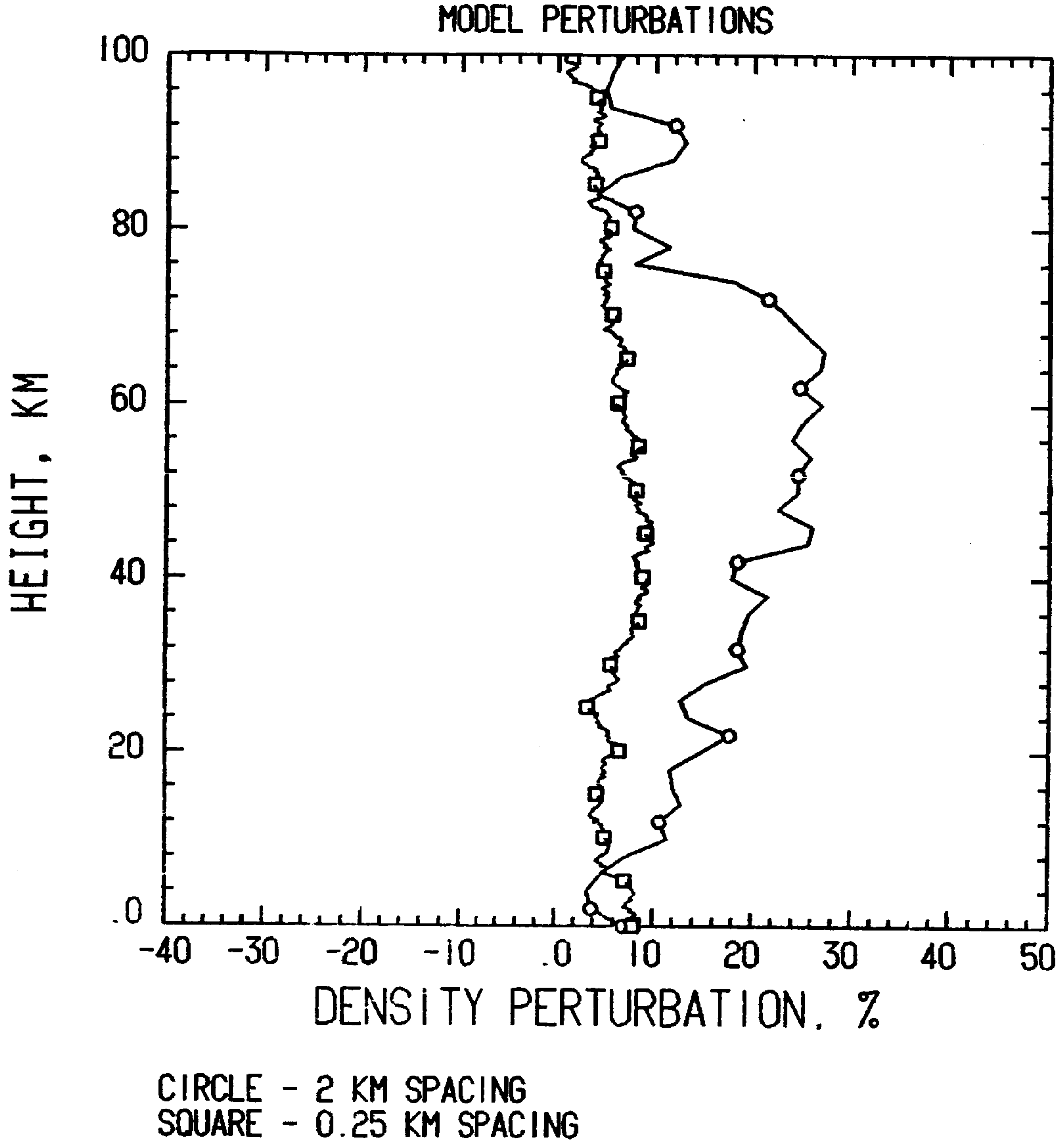

Figure 4 - Simulation of vertical profile of density perturbations with the GRAM two-scale model and the Gaussian correlation function, with two different vertical spacings. Parameter values same as in Figure 1.

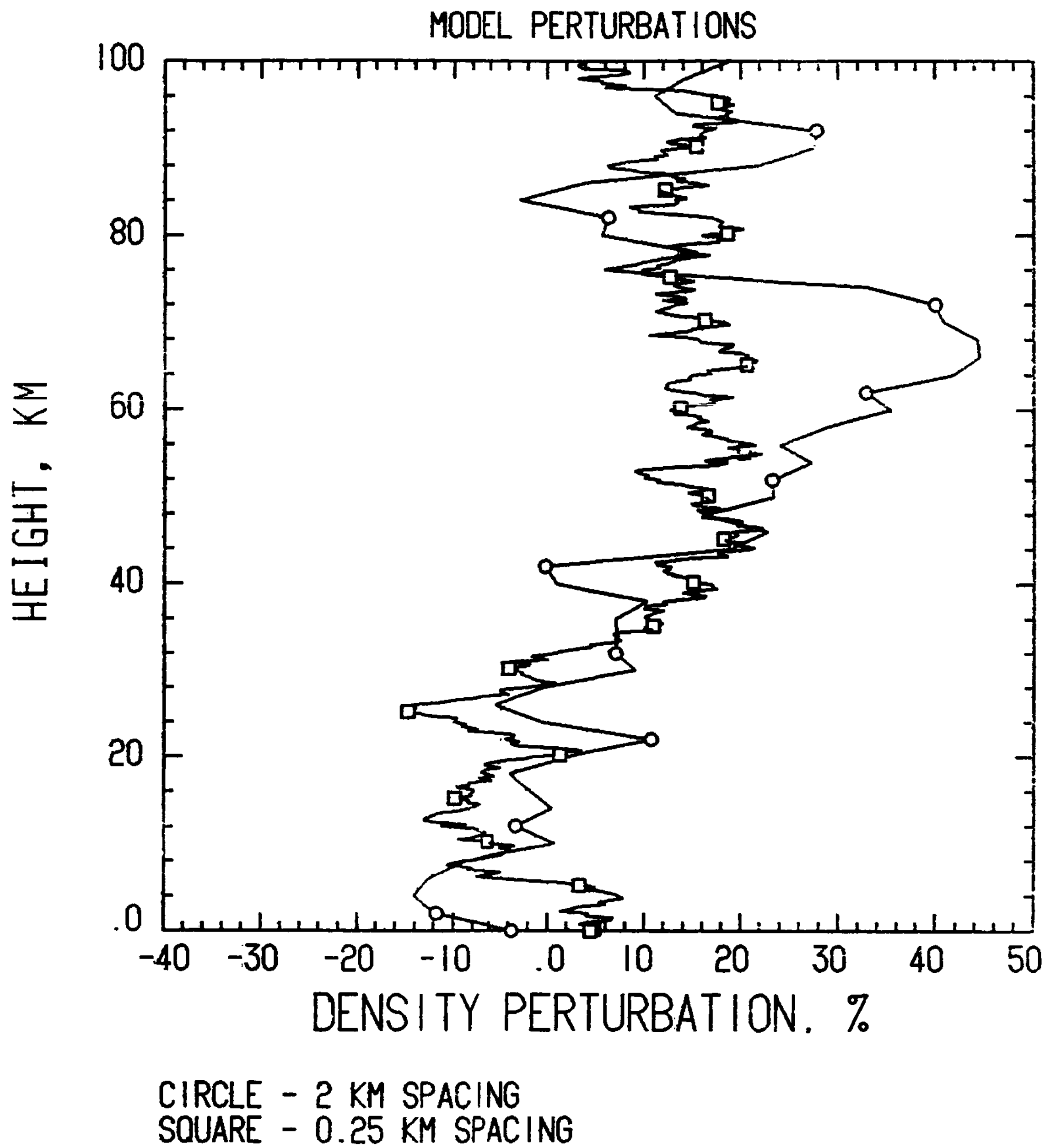

Figure 5 - Simulation of vertical profile of density perturbations with the GRAM two-scale model and the modified exponential correlation function, with two different vertical spacings. Parameter values same as in Figure 1.

Table 1 - Computed values of $\sigma(f)$ (nominal = 10.0) and rms value of gradient, df/dr (nominal given in Table 2), for first and second order autoregressive models [AR(1) and AR(2)], for various correlation functions, and for the fractal model. Dashes indicate no results due to violation of the stationarity constraints in the AR(2) model. Theoretically only the exponential correlation satisfies stationarity for the AR(1) model.

CORRELATION:		EXPONENTIAL		MODIFIED EXP.		GAUSSIAN		PARABOLIC		$\sin(k\Delta x)/(k\Delta x)$		$(1+m\Delta x)\exp(-m\Delta x)$	
$\Delta r/L$	Model	$\sigma(f)$	df/dr	$\sigma(f)$	df/dr	$\sigma(f)$	df/dr	$\sigma(f)$	df/dr	$\sigma(f)$	df/dr	$\sigma(f)$	df/dr
0.20	AR(1)	9.3±1.1	30.8±1.4	9.3±1.1	30.8±1.4	6.4±1.6	12.7±0.5	5.3±1.5	9.6±0.3	5.2±1.5	9.2±0.3	7.7±1.6	17.9±0.7
	AR(2)	9.3±1.1	30.8±1.4	9.3±1.1	30.8±1.4	7.6±2.2	9.8±2.4	–	–	4.3±1.5	4.2±1.0	9.1±1.3	17.9±1.4
	Fractal	8.7±3.0	14.7±2.7										
0.10	AR(1)	8.5±1.8	44.3±2.0	8.5±1.8	44.3±2.0	5.4±2.9	12.6±0.6	4.5±2.5	9.5±0.4	4.4±2.5	9.2±0.4	6.5±3.0	18.9±0.9
	AR(2)	8.5±1.8	44.3±2.0	8.5±1.8	44.3±2.0	5.8±2.3	7.6±2.7	–	–	3.3±1.5	3.2±1.2	8.2±2.1	18.1±1.5
	Fractal	9.1±2.7	17.0±2.2										
0.08	AR(1)	7.9±1.8	49.7±2.0	7.9±1.8	49.7±2.0	4.1±1.8	12.6±0.5	3.3±1.4	9.5±0.4	3.1±1.4	9.1±0.4	5.4±2.1	19.1±0.8
	AR(2)	7.9±1.8	49.7±2.0	7.9±1.8	49.7±2.0	3.9±1.5	5.3±1.9	–	–	2.0±0.7	1.8±0.4	7.2±2.1	17.1±1.3
	Fractal	10.1±2.7	17.4±3.6										
0.06	AR(1)	9.1±2.0	57.8±2.1	9.1±2.0	57.8±2.1	4.5±1.6	12.7±0.5	3.5±1.3	9.6±0.4	3.4±1.2	9.2±0.3	6.0±1.9	19.5±0.7
	AR(2)	9.1±2.0	57.8±2.1	9.1±2.0	57.8±2.1	5.4±2.0	6.9±2.3	–	–	2.8±1.3	2.7±1.0	9.0±2.3	18.5±2.2
	Fractal	9.5±2.3	17.8±2.0										
0.04	AR(1)	8.9±2.0	69.6±1.6	8.7±2.0	62.1±1.4	3.6±1.7	12.4±0.3	2.8±1.4	9.4±0.2	2.7±1.3	9.0±0.2	5.1±2.2	19.3±0.4
	AR(2)	8.9±2.0	69.6±1.6	9.0±1.9	62.4±1.8	5.6±2.5	7.1±3.0	–	–	2.7±1.3	2.5±1.0	8.8±2.6	18.3±2.7
	Fractal	10.2±2.7	17.8±2.9										
0.02	AR(1)	8.2±1.2	100.0±2.1	7.3±1.3	62.7±1.1	2.6±0.8	12.6±0.3	2.0±0.7	9.5±0.2	1.9±0.6	9.1±0.2	3.8±13	19.8±0.4
	AR(2)	8.2±1.2	100.0±2.1	–	–	3.1±0.9	3.9±1.0	–	–	–	–	7.9±1.3	17.4±1.9
	Fractal	8.2±2.6	22.2±5.9										

Table 2 - Expected values of rms gradient $df/dr = (\Delta f/\Delta r)_{rms}$ from equation (5) and the various correlation function models used in Table 1.

CORRELATION: $\Delta r/L$	EXPO-NENTIAL	MODIFIED EXP.	GAUS-SIAN	PARA-BOLIC	$\sin(k\Delta x)/(k\Delta x)$	$(1+m\Delta x)\times\exp(-m\Delta x)$
0.20	30.11	30.11	12.44	9.43	9.05	17.54
0.10	43.63	43.63	12.51	9.43	9.06	18.72
0.08	49.02	49.03	12.52	9.43	9.07	18.97
0.06	56.88	56.89	12.52	9.43	9.07	19.22
0.04	70.01	62.48	12.53	9.43	9.07	19.48
0.02	99.50	62.48	12.53	9.43	9.07	19.74